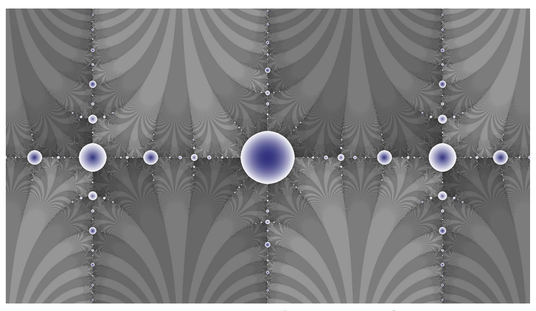
ORBITS: Making Music with Mathematics

In an innovative collaboration between the Department of Mathematical Sciences and leading composer Dr Emily Howard, ground-breaking mathematical research conducted at Liverpool is inspiring and influencing new music.



A structure arising in dynamical system research at Liverpool.

The Collaboration

Since 2015, the Department of Mathematical Sciences has been collaborating with award-winning composer **Dr Emily Howard**. From February to November 2015, this partnership was funded by the Leverhulme Trust through an **Artist in Residence** award, the first such project hosted at the University of Liverpool. Throughout this period, Dr Howard visited regularly for wide-ranging discussions on topics of current research, both in the department and across the wider University.

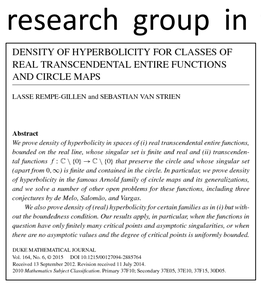
The innovative collaboration pairs a leading composer with a background in mathematics with a leading mathematics department eager to engage with art and artists. It has led to several new pieces of music, inspired directly or indirectly by mathematical research, as well as a series of public events and performances taking place both nationally and internationally.

Emily Howard and Lasse Rempe-Gillen discussing *Orbits*.

Dynamical Systems

A **dynamical system** is a system that changes over time according to a fixed rule. Even simple rules can give rise to highly complicated, **"chaotic"** behaviour, where tiny changes in the starting condition can, over time, lead to drastically different long-term behaviour.

The Department of Mathematical Sciences hosts a world-leading research group in the area. During her residency, Dr Howard held many discussions with these researchers, particularly concerning a recent breakthrough by **Prof. Lasse Rempe-Gillen** (Liverpool) and **Prof. Sebastian van Strien** (Imperial College). This work, published in 2015, solved a long-standing open problem dating back to the 1960s.



DENSITY OF HYPERBOLICITY FOR CLASSES OF REAL TRANSCENDENTAL ENTIRE FUNCTIONS AND CIRCLE MAPS

LASSE REMPE-GILLEN and SEBASTIAN VAN STRIEN

Abstract

We prove density of hyperbolicity in spaces of (i) real transcendental entire functions, bounded on the real line, whose singular set is finite and real and (ii) transcendental functions $f : \mathbb{C} \setminus \{0\} \to \mathbb{C} \setminus \{0\}$ that preserve the circle and whose singular set (apart from 0, ∞) is finite and contained in the circle. In particular, we prove density of hyperbolicity in the famous Arnold family of circle maps and its generalizations, and we solve a number of other open problems for these functions, including three conjectures by de Melo, Salomão, and Vargas.

We also prove density of (real) hyperbolicity for certain families as in (i) but without the boundedness condition. Our results apply, in particular, when the functions in question have only finitely many critical points and asymptotic singularities, or when there are no asymptotic values and the degree of critical points is uniformly bounded.

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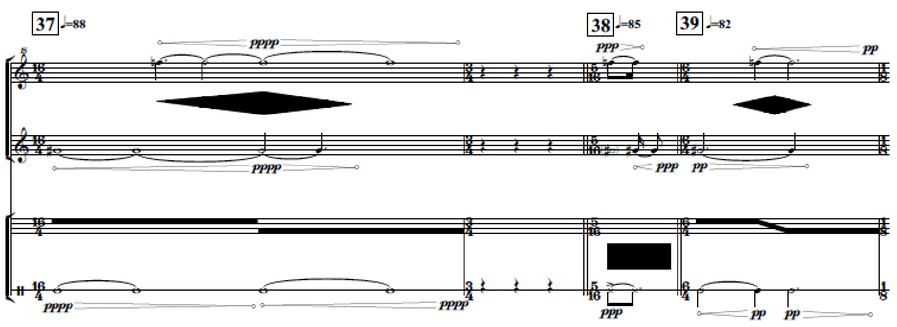
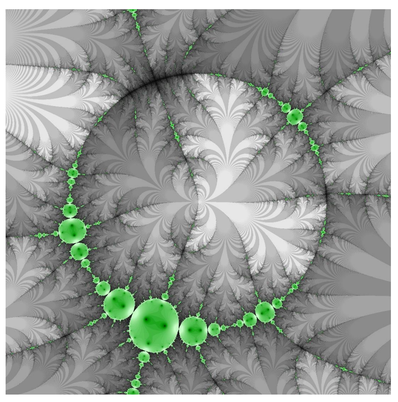


An excerpt of *Leviathan*.



Emily Howard and Lasse Rempe-Gillen (right) with tubist Jack Adler-McKean ahead of a performance of *Chaos or Chess*.

Composer Emily Howard

Dr Howard

Dr Emily Howard is an award-winning British composer. Her music is commissioned, performed, and broadcast internationally by festivals and ensembles including the BBC Proms, New Scientist Live, Wien Modern, the London Symphony Orchestra and Bamberger Symphoniker.

In addition to Masters and doctoral degrees in composition from the Royal Northern College of Music and Manchester University, Dr Howard holds an undergraduate degree in mathematics and computer science from Oxford University. Her music is known for its particular connection to **science and mathematics**.

Data from the theory of dynamical systems, used for Orbit 1a (top) and Orbit 2a (bottom).

Composition

Insights and ideas from current mathematical research absorbed during conversations with Liverpool researchers provide input into Dr Howard's compositional process in a variety of ways:

Leviathan (2015)

In *Leviathan* (2015), baritone saxophone and percussive sounds collide in the creation of an **unpredictable "chaotic" musical language** that responds to ideas from recent mathematical research in dynamical systems. *Leviathan* was performed by the duo **scapegoat** on a US tour in 2015 and is recorded by them on Dr Howard's NMC Debut Disc "Magnetite".

Orbits (2015-16)

The series of chamber works entitled "Orbits" (2015-2016) is a direct response to research of Rempe-Gillen and van Strien, a **translation of mathematical ideas into sound**, with musical processes and structures informed by their work on the density of hyperbolicity for the Arnold family of dynamical systems on the circle. *Orbit 1a* was performed and broadcast by RNCM students as part of Dr Howard's **Proms Extra** Composer Conversation (BBC Proms 2016).

Chaos or Chess (2016)

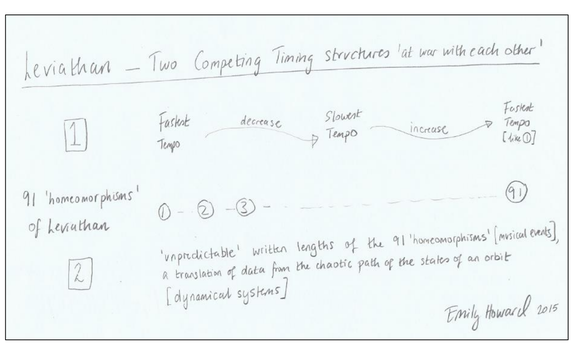
A further "Orbit" chamber work *Chaos or Chess* (2016) for solo microtonal tuba plays more freely with some of the mathematical ideas in the earlier "Orbits". *Chaos or Chess* was performed and broadcast by **Jack Adler-McKean** as part of Dr Howard's Proms Extra Composer Conversation and has since been toured in the US (02/2017).

Torus (Concerto for Orchestra) (2016) and sphere (2017)

Large-scale works *Torus (Concerto for Orchestra)* (2016) and *sphere* (2017) – both named after mathematical objects – also contain and interpret modern mathematical ideas freely, alongside other influences. *sphere* was commissioned by the **Bamberger Symphoniker** and premiered in Germany (03/2017) while *Torus (Concerto for Orchestra)* was co-commissioned by the **BBC Proms** and **Royal Liverpool Philharmonic** and premiered by the Royal Liverpool Philharmonic Orchestra conducted by Vasily Petrenko at the BBC Proms 2016.

Ongoing Collaborations

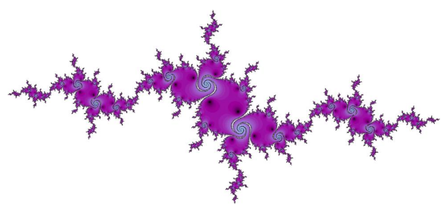
The Department of Mathematical Sciences and Dr Howard remain in close contact, and the **Orbits** collaboration with the Dynamical Systems group is ongoing. In addition, the new collaboration **Music and Mathematics interrogate brain tumour dissemination** promises exciting interactions with the recently established **Liverpool Centre for Mathematics and Healthcare** at the University of Liverpool.



Emily Howard's handwritten notes on the timing of *Leviathan*.

https://www.liverpool.ac.uk/mathematical-sciences/artist-in-residence/

Emily Howard (second from the left) talking to members of staff of the Department of Mathematical Sciences.



PRiSM

Dr Howard is the founder and director of PRiSM, the **RNCM Centre for Practice & Research in Science & Music**, alongside her co-director, mathematician Prof. Marcus du Sautoy. Inspired in part by her residency at the Department of Mathematical Sciences, PRiSM brings together a number of **creative collaborations** between the sciences and music – including the ongoing collaboration with the Department of Mathematical Sciences.

www.rncm.ac.uk/prism

scapegoat (Noam Bierstone (front) and Joshua Hyde) rehearsing *Orbits*.

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PRiSM

RNCM ROYAL NORTHERN COLLEGE of MUSIC

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